Neutrino mass characteristics in a phenomenological $3 + 2 + 1$ model

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(Dated: September 5, 2012)

The basic characteristics used for the description of both Dirac and Majorana massive neutrinos are studied. Currently available experimental data for these characteristics are presented with an evidence of possible anomalies beyond the Standard Model with three light neutrinos. Special attention has been paid to ways to determine the neutrino mass absolute scale. In accordance with the available data, the permissible values of the neutrino mass matrix elements are found numerically against the minimal mass of neutrino. Some phenomenological relations for the masses, the angles and the *CP*-violating phases of the neutrino mixing matrix are discussed, and the values of the neutrino mass characteristics and the Dirac *CP*-violating phase are evaluated for a model of bimodal neutrinos. The estimations made for masses of sterile neutrinos, the values of the neutrino mass characteristics and the Dirac *CP*-violating phase can be used for interpretation and prediction of the results of various neutrino experiments.

PACS numbers: 14.60.Pq, 14.60.St, 12.10.Kt, 12.90.+b
Keywords: neutrino oscillations; mixing angle; *CP*-violating phase; neutrino mass; neutrino observable; sterile neutrino; bimodal neutrinos

I. INTRODUCTION

During the past years, a number of new interesting experimental results were obtained in the neutrino physics. To such results, one can refer the deviation from zero (more than 5σ) of the reactor mixing angle θ_{13} [1] and the experimental indications on possible existing of new anomalies for the number of neutrinos and antineutrinos in different processes [2]. The first result is important for determination of both the neutrino mixing matrix and the neutrino mass matrix [3], and also for justification of the experimental search of *CP* violation in the lepton sector [4]. As is known, the experimental data confirming lepton CP violation are absent to the present. The second result can be connected with existence of light sterile neutrinos, to which one can refer, for example, $SU(2)_L$ singlet right neutrinos. The existence of right neutrinos is beyond the Standard Model (SM) of weak and electromagnetic interactions. However, the discovery of oscillations of atmospheric, solar, reactor and accelerator neutrinos, as well as extremely smallness of the neutrino mass definitely points to violation of the conservation laws of the leptonic numbers L_e, L_μ and L_τ and existence of right neutrinos.

One of the unsettled outstanding problem in the neutrino physics is the question about the neutrino nature, that is the question, does neutrino belong to the Dirac or the Majorana particle type? These two types of elementary particles were introduced in the particle physics in 1928 by Dirac [5] and in 1937 by Majorana [6], respectively. In the present, considerable efforts by both experimentalists and theorists are concentrated to solve this problem. In this, it can be unexpected the lack of unambiguous answer. For example, neutrinos can simultaneously posses by both the Dirac and the Majorana properties [7–10]. One of the models, in which such situation is realized, is the model of bimodal (*schizophrenic*) neutrinos [11–13].

The other question requiring its speedy solution is the question about the absolute scale of neutrino masses. As is known, from the oscillation experiments with neutrinos it is possible to determine the absolute values of the differences of the neutrino mass in squares, rather than the mass values themselves. This circumstance gives rise to the problem of the neutrino mass hierarchy, as well as to the problem of the absolute mass scale. In spite of these problems can be distinctly solved only with using the results of the future special experiments, the different variants of their solutions in the framework of the phenomenological models are also proposed [14–17].

In the current paper, a phenomenological $3+2+1$ neutrino model is proposed for solution of the problems in the neutrino physics noted above. This model involves three active light neutrinos and also three sterile neutrinos, from which one sterile neutrino is comparatively heavy, but two other are light sterile neutrinos [18]. It is important that light sterile neutrinos may be practically degenerate by mass with light active neutrinos, so that they can jointly form quasi Dirac neutrinos, thus realizing the bimodal neutrino states with two Majorana and two quasi Dirac neutrinos. On the basis of available experimental data, the acceptable ranges for neutrino mass characteristics in this model are found numerically. Allowing for the cosmological restrictions on total number of neutrinos [19], it is easy to reduce the number of sterile neutrinos in the framework of the model under consideration, that is to pass from the $3 + 2 + 1$ model to the $3 + 1 + 1$, or even the $3 + 1$ model. However, such a reduction should be caused by forcible reasons. In this connection, note that the experimentally obtained cosmolog-

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ical restrictions on total number of neutrinos in turn are model-dependent, and additional studies of the models permitting existence of five or even six Majorana neutrinos, as well as more accurate experimental restrictions on the number of neutrinos are indispensable.

The paper is organized as follows. In Sec. II, the basic characteristics used for the description of both the Dirac and the Majorana massive neutrinos, as well as available experimental data for these characteristics are present. For the latter, the possible anomalies beyond the SM with three light active neutrinos (with masses less than $m_Z/2$, where m_Z is the mass of the Z–boson) are indicated. In Sec. III, a special attention is paid to ways of determination of the neutrino mass absolute scale, and also to restrictions on this scale following from the experimental data. In Secs. IV and V, respectively, some phenomenological relations for the neutrino masses, as well as for the mixing angles and the CP-violating phases of the neutrino mixing matrix are discussed, and the values of the neutrino mass observables and the Dirac CP-violating phase for the 3+2+1 neutrino model are calculated. The results of the paper, which can be used for the interpretation and prediction of the results of various neutrino experiments are discussed in Sec. VI, which terminates the paper.

II. EXPERIMENTAL VALUES OF THE BASIC CHARACTERISTICS OF THE DIRAC AND MAJORANA MASSIVE NEUTRINOS

It is known that the oscillations of the solar, atmospheric, reactor and accelerator neutrinos can be explained by mixing the neutrino states. It means that the flavor states of neutrino are the mix, at least, of three massive neutrino states, and vice versa. Neutrino mixing is described by the Pontecorvo–Maki–Nakagawa–Sakata matrix $U_{PMNS} \equiv U = VP$, which enters the relation

$$\psi_{\alpha L} = U_{\alpha i}\psi_{iL}, \tag{1}$$

where $\psi_{\alpha L}$ and ψ_{iL} are the left chiral flavor or massive neutrino fields, respectively, with $\alpha = \{e, \mu, \tau\}$ and $i = \{1, 2, 3\}$, and summation over repeated indices is implied. The matrix V can be written in the standard parameterization [20] as

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix}, \tag{2}$$

through the quantities $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$, and with the phase δ associated with the Dirac CP violation in the lepton sector. On the other hand, the 3×3 matrix P is the diagonal one, $P = \text{diag}\left\{1, e^{i\alpha}, e^{i\beta}\right\}$, with α and β the phases associated with the Majorana CP violation.

Generally, an unitary $n \times n$ matrix is defined by n^2 real parameters, which can be chosen as $n(n-1)/2$ angles and $n(n+1)/2$ phases. Taking into account the structure of the SM electro-weak lagrangian involving currents of quarks, charged leptons and neutrinos, in the case of the Dirac neutrinos it is possible to exclude $2n-1$ phases from these parameters. In the opposite case, if all the neutrinos are the Majorana type particles, it is possible to exclude only n phases associated with the Dirac charge leptons. Thus, depending on the neutrino type, the $n \times n$ U_{PMNS} matrix is defined by either $n(n-1)/2$ angles and $(n-1)(n-2)/2$ phases, if neutrinos belong to the Dirac type, or $n(n-1)/2$ angles and $n(n-1)/2$ phases, if neutrinos are the Majorana-type particles [21].

With the help of the neutrino mixing matrix U, the mass matrix M of three active neutrinos can be determined as follows

$$M = U^* M_d U^+ \,, \tag{3}$$

where $M_d = \text{diag}(m_1, m_2, m_3)$, with m_i the neutrino masses. Matrix elements M_{ij} of the matrix M depend on the neutrino masses and the mixing parameters, that is the mixing angles and phases. For the case of three light Dirac neutrinos, in addition to masses it is necessary to determine three mixing angles (θ_{12}, θ_{13} and θ_{23}) and one mixing phase (CP-violating phase δ), while for three Majorana neutrinos it requires to determine three mixing angles (θ_{12}, θ_{13} and θ_{23}) and three CP-violating phases (δ, α and β).

The determination of the neutrino mass absolute values and the mixing parameters, as well as ascertainment of the neutrino type (Dirac vs Majorana) are at present the basic problems of the neutrino physics. Solutions of these problems require active experimental and theoretical studies of both the neutrino mass observables, which define the absolute mass scale, and the neutrino oscillation characteristics, which characterize mixing of the neutrino states with different masses. It is expected that the comprehensive solution of the problem of the neutrino nature, as well as of the neutrino masses and mixing parameters will be given in the future Grand Unification Theory (GUT), which does not exist at present in its conventional form [20]. A required direction of the further development of the SM can also be found by study of different correlations between experimental values of the neutrino masses and mixing parameters, to which we refer the angles of mixing and the CP-violating phases.

The discovery of such interrelationships can play an important role in finding out the ways to expand the SM and to successfully develop at last the following consistent GUT. For this, the data received in the current and the future experiments on determination the neutrino characteristics (PLANK, KATRIN, GERDA, CUORE, BOREXINO, Double CHOOZ, SuperNEMO, KamLand-Zen, EXO, etc.) will undoubtedly play a decisive role.

With using the only oscillation experiments with atmospheric, solar, reactor and accelerator neutrinos it is impossible to determine the neutrino mass absolute values, as well as the type, either Majorana or Dirac, of the neutrino. However, the experimental data obtained in the neutrino oscillation experiments indicate the violation of the conservation laws of the leptonic numbers L_e, L_μ, L_τ, and, besides, by virtue of deviation from zero of two oscillation parameters Δm_{12}^2 and Δm_{13}^2 (with $\Delta m_{ij}^2 = m_i^2 - m_j^2$) they indicate the existence at least two nonzero and different neutrino masses. Below we present the experimental values of the mixing angles and the oscillation parameters, which determine three-flavor oscillations of the light neutrinos. Together with the standard uncertainties on the level of 1σ, these data obtained as a result of a global analysis of the latest high-accuracy measurements of the oscillation parameters [1] are as follows

$$\sin^2\theta_{12} = 0.307^{+0.018}_{-0.016}\,, \tag{4a}$$

$$\sin^2\theta_{23} = \begin{cases} NH: & 0.386^{+0.024}_{-0.021} \\ IH: & 0.392^{+0.039}_{-0.022} \end{cases}, \tag{4b}$$

$$\sin^2\theta_{13} = \begin{cases} NH: & 0.0241^{+0.0025}_{-0.0025} \\ IH: & 0.0244^{+0.0023}_{-0.0025} \end{cases}, \tag{4c}$$

$$\Delta m_{21}^2/10^{-5}\mathrm{eV}^2 = 7.54^{+0.26}_{-0.22}\,, \tag{4d}$$

$$\Delta m_{31}^2/10^{-3}\mathrm{eV}^2 = \begin{cases} NH: & 2.47^{+0.06}_{-0.10} \\ IH: & -2.46^{+0.07}_{-0.11} \end{cases}. \tag{4e}$$

Since only the absolute value of Δm_{31}^2 is known, it is possible to arrange the absolute values of the neutrino masses by two ways, namely, as

$$a)\ m_1 < m_2 < m_3 \quad \text{and} \quad b)\ m_3 < m_1 < m_2\,. \tag{5}$$

These two cases correspond to so called the normal hierarchy (NH) and the inverse hierarchy (IH) of the neutrino mass spectrum, respectively. Unfortunately, the CP-violating phases α, β and δ, as well as the neutrino mass absolute scale are unknown at present.

Three groups of the experimental data associated with the neutrino are sensitive to the neutrino absolute mass scale, namely, they are the data on β decay, the data on neutrinoless double-beta decay, and the data obtained as a result of the cosmological observations. So, to determine the neutrino absolute mass scale it is necessarily to determine experimentally at least one from three mass observables of the neutrino, namely, either the mean cosmological mass m_a of the active neutrinos, or the β decay neutrino mass m_β, or the effective double-beta decay neutrino mass $m_{\beta\beta}$, which are defined as follows

$$m_a = \frac{1}{3}\sum\nolimits_{i\,=\,1,2,3} |m_i|\,, \tag{6a}$$

$$m_\beta = \Big(\sum\nolimits_{i\,=\,1,2,3} |U_{ei}|^2 m_i^2\Big)^{1/2}, \tag{6b}$$

$$m_{\beta\beta} = \Big|\sum\nolimits_{i\,=\,1,2,3} U_{ei}^2\, m_i\Big|\,, \tag{6c}$$

where U_{ei} are the elements of the Pontecorvo–Maki–Nakagawa–Sakata matrix. Generally, we will call both the neutrino mass observables and the neutrino masses as the mass characteristics of the neutrino. With help of the corresponding data from these three groups of the experiments indicated above, three mass observables m_a, m_β, and $m_{\beta\beta}$ one by one can be determined, respectively. Currently, the upper limits are only obtained for the mass observables, namely, $m_a < 0.2$ eV [19], $m_\beta < 2.2$ eV [22], and $m_{\beta\beta} < 0.35$ eV [23, 24], where in the last case the limit should be increased approximately one and a half or even twice to take into account the uncertainties in the nuclear matrix elements governing this limitation. Note that the limit for m_β given above, which was obtained in the experiments carried out in Troitsk and Mainz on the electron spectrum measurements in the tritium β decay, is planned to be improved up to 0.2 eV in the scheduled KATRIN experiment [22].

As we know, the right neutrinos are sterile particles and are the candidates, along with the other possibilities, to the specific particles of the dark matter. There can be several types of the dark matter particles. We consider only the right neutrinos ν_R. If ν_R together with the ν_L form the Dirac mass term in the Lagrangian, then ν_R will be degenerate in mass with ν_L thus being the light particles with the mass less than, at least, 0.3 eV. The other possibility is that the masses of ν_R can be larger than the masses of ν_L. For example, they can belong to the range from 0.3 eV up to 3 eV. The right neutrinos of such a kind can be called as heavy sterile neutrinos. Besides, the neutrinos with masses from 3 eV up to 3 GeV and with masses more than 3 GeV can be called as extra-heavy and super-heavy neutrinos, respectively. The existence of super-heavy right neutrinos can be used both for explanation of high value of the invisible mass of the Universe, and for the explanation of small masses of the left neutrinos. Moreover, with the super-heavy right neutrinos, the observed baryon asymmetry of the Universe can be explained [25].

Recently [26], the corrected calculations of the spectrum of reactor antineutrino were provided, which result in higher calculated values of the fluxes of these particles. Thus, the experimental data indicate on the antineutrino deficiency in the measurements of the antineutrino fluxes on distances lower than 100 m from the particle source. These distances from the source should be considered as small. The currently available indications on the antineutrino deficiency on small distances can result in reactor anomaly. Rather like anomalies were observed in calibration measurements for the experiments GALLEX and

SAGE [27, 28]. These anomalies can be called as calibration ones. The most recent experimental data indicating that the angle θ_{13} is noticeably deviated from zero [cf. Eq. (4c)] enhance the probability of the reactor and calibration anomalies. In this, the estimation $\sin^2\theta_{14} \lesssim 0.04$ for the mixing angle θ_{14} of the active and sterile neutrinos is obtained in the $3_a + 1_s$ model with single sterile neutrino [18, 29]. Note that, besides the $3_a + 1_s$ model, the $3_a + 2_s$ model with three active and two sterile neutrinos is used for explanation of the observed anomalies of the spectra of neutrinos and antineutrinos, including the neutrino spectra anomalies in the LSND experiment [30], and then in the MiniBooNE experiment [31] and in the calibration measurements [27, 28]. In these models, the typical values of Δm^2 are of the order of 1 eV^2.

III. EVALUATION OF THE NEUTRINO MASS ABSOLUTE SCALE FROM THE POSSIBLE VALUES OF THE NEUTRINO MASS OBSERVABLES

Let us consider the neutrino mass matrix given by Eq. (3). The upper diagonal matrix element M_{11} of this matrix enters the relation for the probability of the nuclear neutrinoless double-beta decay $(A, Z) \to (A, Z+2) + 2e$, if the decay occurs with the assistance of the Majorana light neutrinos. In this case the absolute value of M_{11} coincides with $m_{\beta\beta}$ from Eq. (6c), i.e., with the neutrino effective mass. The nuclear half-life period $T_{1/2}^{0\nu\,2\beta}$ is inversely proportional to $m_{\beta\beta}^2$ [32]. Note that the discovery of the neutrinoless double-beta decay is practically the only way to determine whether neutrinos are of the Dirac-type or the Majorana-type particles. The discovery of this decay could make it possible to determine the neutrino absolute mass scale with the aid of the $m_{\beta\beta}$ experimental value.

The explicit expression for $m_{\beta\beta}$ through the neutrino masses and the matrix elements of the neutrino mixing matrix U_{PMNS} is given in Eq. (6c). In the present, on the basis of the experimental data given in Eqs. (4) on the mixing parameters of neutrino oscillations, and with the adjusting values of the $m_{\beta\beta}$, it can be possible to estimate the absolute scale of the neutrino mass spectra with the normal and inverse hierarchy. Indeed, $m_{\beta\beta}$ is expressed trough the neutrino masses and the neutrino mixing parameters as follows

$$m_{\beta\beta} = |c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{2i\alpha} m_2 + s_{13}^2 e^{2i(\beta-\delta)} m_3| \,. \quad (7)$$

With using Eq. (7) and the experimental data from Eqs. (4), the explicit dependences of $m_{\beta\beta}$ versus the minimal neutrino mass m_0, that is either versus m_1 in the NH case or versus m_3 in the IH case can be determined. Besides, the similar dependences can be found for absolute values of other matrix elements M_{ij}. Figure 1(a) exhibits the dependences of $m_{\beta\beta}$ versus m_0 jointly for both NH and IH cases at zero value of the neutrino

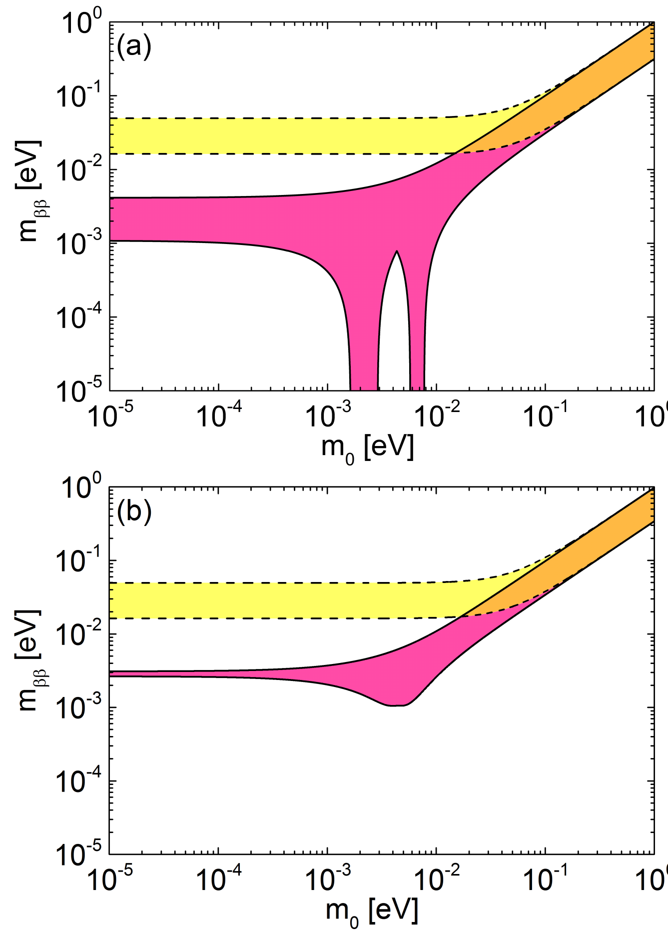
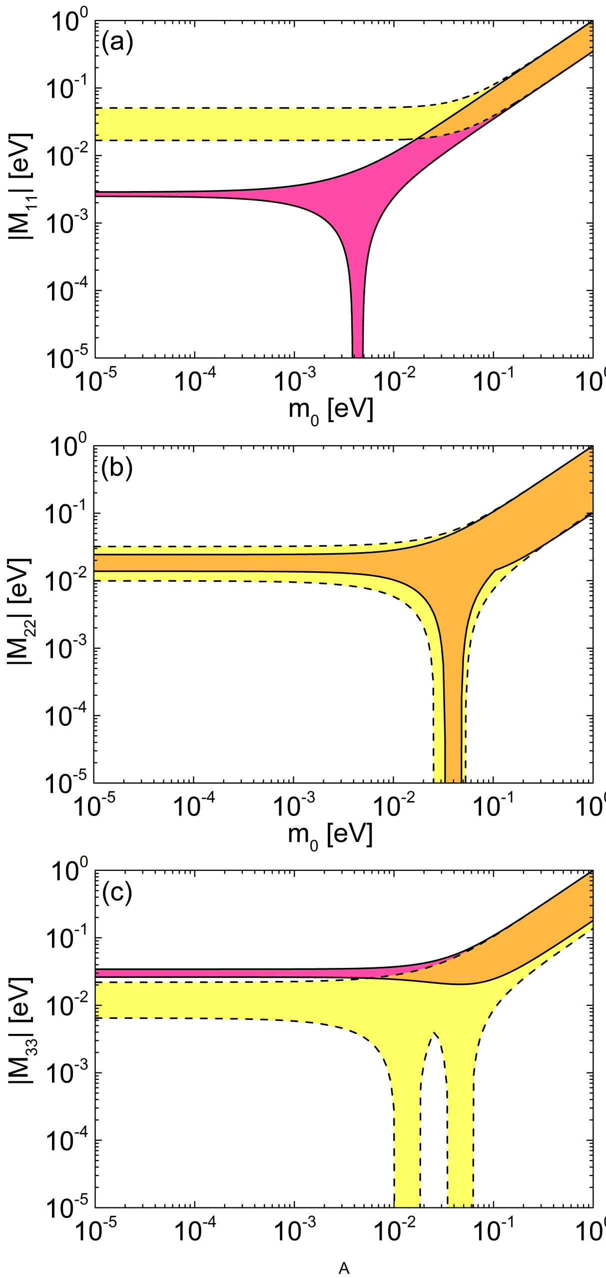


FIG. 1. (Color online) The variation ranges of the effective neutrino mass $m_{\beta\beta}$ as a function of the minimal neutrino mass m_0 at $\delta = 0$ (a) and at $\delta = 45^\circ$ (b), at $\alpha = 0$, $\beta = 0$ and at the values of the oscillation parameters from Eqs. (4). The shaded areas restricted by the solid and the dashed curves correspond to the NH and IH cases, respectively.

CP-violating phases. The allowance for the nonzero *CP*-violating phases alters the valid (from the point of view of available experimental data) range of variation of $m_{\beta\beta}$ versus m_0. For example, the characteristic limitation of the $m_{\beta\beta}$ values from the bottom arises at some values of the *CP*-violating phase δ. Such behavior of $m_{\beta\beta}$ for both NH and IH cases and for $\delta = 45^\circ$ is presented in Fig. 1(b).

It is interesting that increase of accuracy of the numerical calculations together with the new experimental data results in division of the range of the minimal neutrino mass in the NH case on two sub-ranges, where zeroth values of $m_{\beta\beta}$ can be reached. At values of m_0 between these two sub-ranges, $m_{\beta\beta}$ is restricted from the bottom. These new features of behavior of $m_{\beta\beta}$ as a function of m_0 were not noted previously.

The possible structure of the neutrino mass matrix, as well as applicability of some additional assumptions about values of its matrix elements, which are used in



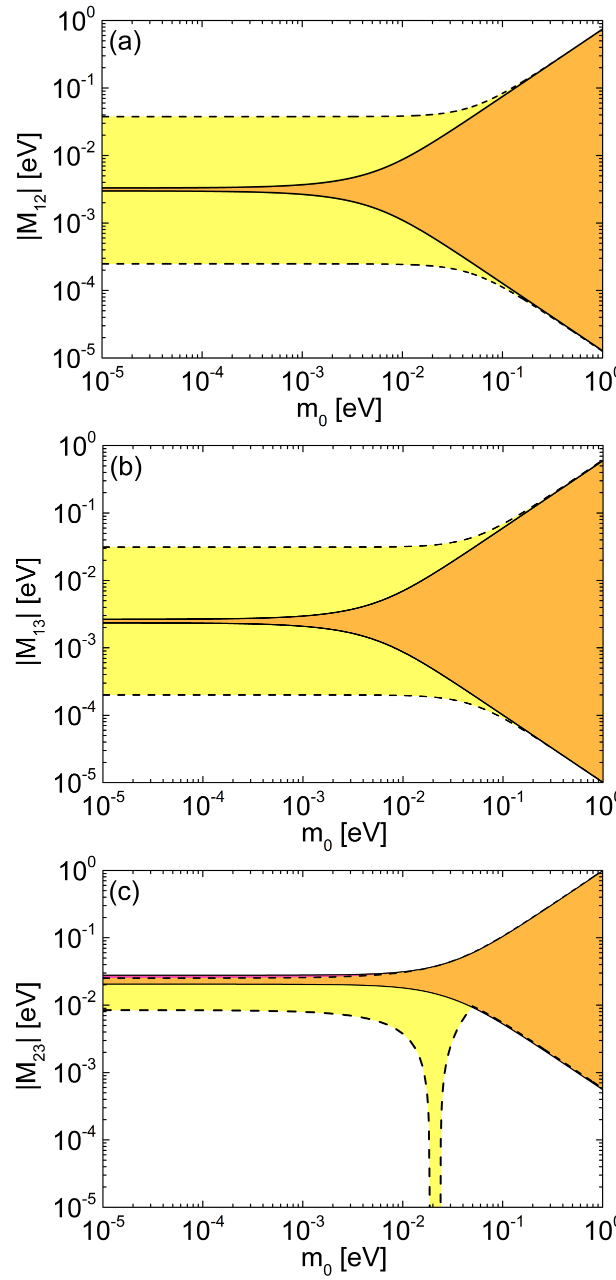


FIG. 2. (Color online) The ranges of absolute values of the matrix elements M_{11} (a), M_{22} (b) and M_{33} (c) versus the minimal neutrino mass m_0 at $\sin^2\theta_{13} = 0$, $\delta = 0$, $\alpha = 0$, $\beta = 0$. The values of the other oscillation parameters coincide with the data from Eqs. (4). The shaded areas restricted by the solid and the dashed curves correspond to the NH and IH cases, respectively.

FIG. 3. (Color online) The same as in Fig. 2, but for off-diagonal matrix elements M_{12} (a), M_{13} (b) and M_{23} (c) of the neutrino mass matrix.

a number of phenomenological models [33, 34], can be obtained numerically. For calculations of the absolute values of the matrix elements M_{ij}, the approximation of $s_{13} = 0$ can be safely used. Figures 2 and 3 exhibit the absolute values of diagonal and off-diagonal elements M_{ij}, respectively, versus the minimal neutrino mass m_0 for both NH and IH cases. Note that in the applied approximation of $s_{13} = 0$, the main features of behavior of $|M_{ij}|$ remain invariable, but the fine peculiarities such a division of the m_0-range, where $m_{\beta\beta}$ vanishes, on the two sub-ranges disappear (cf. Fig. 1(a) and Fig. 2(a) for $|M_{11}|$). These dependences permit us to conceive of the structure of mass matrix M_{ij} at different values of m_0. For example, at $m_0 \approx 3$ meV (the mean values of M_{ij}

are also presented in meV)

$$M^{(NH)} = \begin{pmatrix} 2 & 3 & 2 \\ 3 & 15 & 14 \\ 2 & 14 & 12 \end{pmatrix}, \quad M^{(IH)} = \begin{pmatrix} 30 & 3 & 2 \\ 3 & 20 & 10 \\ 2 & 10 & 20 \end{pmatrix}. \quad (8)$$

The different structures of matrix M_{ij} and different assumptions about the values of its matrix elements were considered in different models [33–36]. For instance, the conditions of vanishing of the individual matrix elements, as well as of its spur and determinant were analyzed. Figures 2 and 3 for the absolute values of the matrix elements M_{ij} can be used for inspection of different models of the neutrino mass matrix structure.

As can be seen from Fig. 1, while specifying the permissible values of $m_{\beta\beta}$ it can be possible to determine the eventual intervals of the values of the minimal neutrino mass m_0, and then the absolute values of the other neutrino masses. Note that such calculations of the neutrino mass absolute values are sufficiently precise at small values of $m_{\beta\beta}$, in spite of uncertainties of the experimental data. They are consistent with the results of the papers [37, 38] in the limits of the precision of the presented graphs and the data used, in which the conditions resulting in the values of $m_{\beta\beta}$ greater than 10^{-3} eV were considered. As is seen from Fig. 1(a), the IH spectrum can not be realized for $\delta = 0$ at $m_{\beta\beta} < 0.01$ eV that was noted before in Ref. [37]. Besides, as a result of the latter work, there is the restriction of the NH spectrum at small values of $m_{\beta\beta}$ at some values of δ [cf. with Fig. 1(b)], as well as at $\delta = 0$ in a small intermediate range [cf. with Fig. 1(a)].

In Figures 4(a) and (b) we present the mean cosmological mass m_a of active neutrinos and the β decay neutrino mass m_β, respectively, versus the minimal neutrino mass m_0, which were obtained with the latest experimental data from Eqs. (4) on the oscillation characteristics of the neutrino.

The obtained intervals of the possible values of $m_{\beta\beta}$ can be used for planning the experiments on search of the neutrinoless nuclear double-beta decay and for interpretation of the results obtained with allowance for the CP violation. The neutrino mass observables m_a and m_β can be used for the experiments with the results dependent on the neutrino mass absolute values.

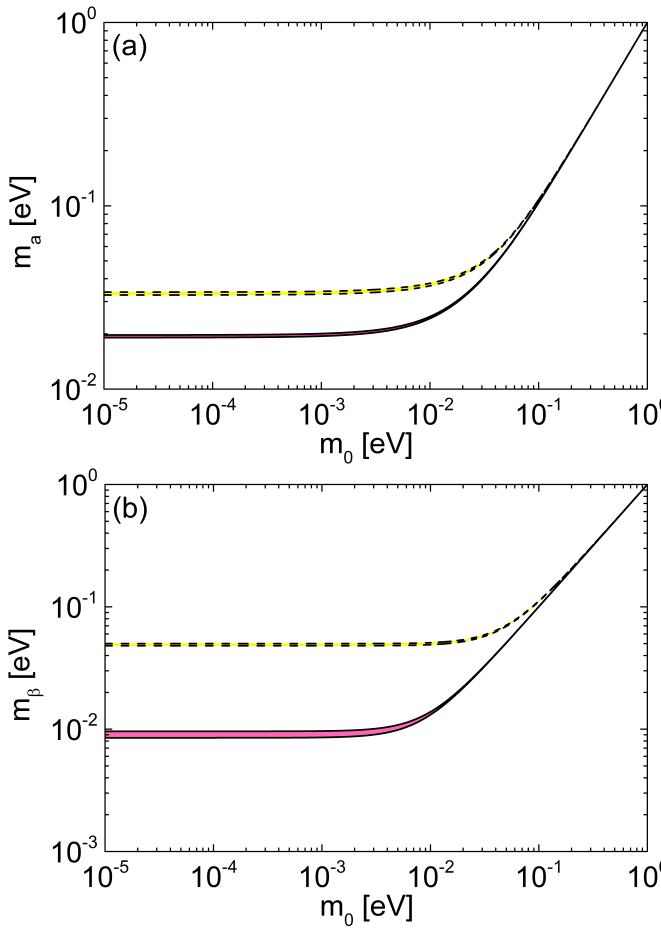


FIG. 4. (Color online) The mean cosmological mass of active neutrinos m_a (a) and the β decay neutrino mass m_β (b) at the oscillation parameters from Eqs. (4). On both panels, the lower and upper narrow ranges restricted by the solid and the dashed curves correspond to the NH and IH cases, respectively.

IV. PHENOMENOLOGICAL RELATIONS FOR THE NEUTRINO MASSES

As is well-known, the problem of origin of the mass of the fundamental fermions is still unsolved. In the SM, these masses are originated due to Yukawa couplings between the fundamental fermion fields and the Higgs fields. Since the actual properties of the Higgs boson are still unknown in detail, the generation mass mechanism accepted in the SM should be considered as a hypothesis. Moreover, the values of the neutrino masses are so small that, probably, the mechanism of the neutrino mass generation is primarily connected with the possible Majorana nature of the neutrino, rather than with the Higgs boson properties. In this case, the immediate basic problem is the simulation of the mass generation mechanism of the Majorana neutrinos. In absence of the appropriate theory of this phenomenon, the problem can be considered on the phenomenological level [17]. Let us suppose that there are several different contributions to the mass of the neutrino, and two of them are most significant. It can be assumed that the first contribution results in the mass of the left light neutrino, which is specified by the Majorana mass term in the Lagrangian as

$$L_m^{'} = -\bar{\nu}_L M_L \nu_L^c/2 + \text{h.c.} \quad (9)$$

In this case, with the Higgs mechanism of the mass generation, the Higgs sector of the SM should be changed and expanded. In what follows, the contribution of the type, which can be associated with Eq. (9), will be taken

into account with the help of a new phenomenological parameter ξ. The second contribution can be connected with the so called seesaw mechanism, which is realized under inclusion into the theory the heavy right neutrinos N_i, with $i = 1, 2, 3$. This mass term is of the form

$$M_\nu^{''} = -M_D^T M_R^{-1} M_D\,, \tag{10}$$

where M_D is the matrix of the Dirac terms in the neutrino masses, M_R is the typical value of the right neutrino masses, which establishes a new scale associated with the right neutrino masses. Let us suppose that M_D is proportional to the mass matrix of the charged leptons, that is $M_D = \sigma M_l$, where σ is of the order of unity, $M_l = diag\{m_e, m_\mu, m_\tau\}$, and $M_R = \sigma^2 M$. Then, for estimations of the neutrino masses m_i, the following phenomenological formula can be used [17]:

$$m_i = \pm\xi - m_{li}^2/M, \tag{11}$$

with m_{li} the masses of three charged leptons. With the help of Eq. (11) and the data from Eqs. (4), it is easily to obtain the absolute values of the neutrino masses m_i and the typical scales of ξ and M in eV for both the NH and the IH cases. They are as follows, respectively,

$$\begin{gathered} m_1 \approx 0.0693,\, m_2 \approx 0.0698,\, m_3 \approx 0.0851, \\ \xi \approx 0.0693,\, M \approx 2.0454 \times 10^{19},\, (NH), \end{gathered} \tag{12a}$$

$$\begin{gathered} m_1 \approx 0.0775,\, m_2 \approx 0.078,\, m_3 \approx 0.0606, \\ \xi \approx 0.0775,\, M \approx 2.2872 \times 10^{19},\, (IH). \end{gathered} \tag{12b}$$

In the mass neutrino generation scheme considered above, the right neutrinos are super-heavy. However, the neutrino mass terms defined by Eq. (9) and Eq. (10) can be given in a different way, if to consider that one of the cause originating the neutrino masses, both for the Majorana and the Dirac neutrinos, is the interaction with the cosmological scalar field of the order of the cosmological Λ-term. Then, both the Majorana masses of the left neutrinos and the Dirac masses may be of the same order with the typical linear mass scale λ of the cosmological Λ-term, it being known that the value of λ is equal approximately 2 meV [19]. In this case, the mass absolute values of three light active neutrinos m_i will be equal in eV for the NH and IH cases, respectively, to

$$m_1 \approx 0.002,\, m_2 \approx 0.0087,\, m_3 \approx 0.0497,\, (NH), \tag{13a}$$
$$m_1 \approx 0.0496,\, m_2 \approx 0.050,\, m_3 \approx 0.002,\, (IH), \tag{13b}$$

while the masses of the right neutrinos M_i can be estimate as follows

$$M_1 \approx \Lambda_1,\, M_2 \approx 0.002,\, M_3 \approx 0.002,\, (NH), \tag{14a}$$
$$M_1 \approx 0.002,\, M_2 \approx 0.002,\, M_3 \approx \Lambda_3,\, (IH), \tag{14b}$$

with $\Lambda_{1,3}$ the free parameters of the order of 1 eV. Note that in this case the possibility exists to identify the right neutrinos ν_{Ri} with the sterile neutrinos ν_{si} $(i = 1, 2, 3)$. This possibility leads to existence of three sterile neutrinos, two of them are light and the third one is heavy. Let us call this case with three light active neutrinos, one heavy sterile neutrino and two light sterile neutrinos as the 3+2+1 model. In case of need, this model can be reduced to the 3+1+1, or even to the 3+1 model with the exception of the light right neutrinos. However, it should be some additional weighty reasons for absence of either one or even two right neutrinos. The light right sterile neutrinos can be combined with light left neutrinos to form the quasi Dirac neutrinos. The case of quasi Dirac neutrinos is considered minutely in the next Sec. V.

V. PHENOMENOLOGICAL RELATIONS FOR THE ANGLES AND THE *CP*-VIOLATING PHASES OF THE NEUTRINO MIXING MATRIX AND THE MODEL OF BIMODAL NEUTRINO

The upper diagonal matrix element $m_{\beta\beta}$ of the neutrino mass matrix is connected with the probability of the neutrinoless double-beta decay. At the same time, two other diagonal matrix elements can be equal in absolute values if the $\mu - \tau$ symmetry is taken into account. It does not contradict to a number of models of the neutrino mass matrix [33, 34] and to the approximate $\mu-\tau$ symmetry [39], as well as to estimations obtained in Sec. III for the matrix elements M_{ij} [see. e.g., Eq. (8)]. However, even in this approximation it is impossible to obtain without additional assumptions the estimations of $m_{\beta\beta}$ in the presence of the *CP* violation[1]. To obtain such estimations, one can consider the case of bimodal neutrino [11–13, 17], when the neutrino is neither the Dirac one nor the Majorana one but has simultaneously both quasi Dirac and Majorana properties. Indeed, a pair of quasi degenerate Majorana neutrinos can form the states of quasi, pseudo, and imaginary Dirac particles. Quasi Dirac particles consist of a pair of one active and one sterile neutrino, pseudo Dirac particles consist of a pair of active neutrinos, and imaginary Dirac particles consist of a pair of sterile neutrinos. Since the investigations of such particles are only starting now, the terminology given above is not completely established. Such particles have the properties of the Dirac particles and become entirely the Dirac particles under full degeneration.

In the model of bimodal neutrino it is usually assumed that the states of neutrino with a certain mass involve both the Majorana and the quasi/pseudo Dirac particles. Let us consider the typical case when, from three active neutrinos, one neutrino is purely Majorana neutrino, while two other ones are quasi Dirac particles. In

[1] Without the *CP* violation, the estimations for $m_{\beta\beta}$ can be easily obtained as 0.07 (NH) and 0.08 eV (IH) at the values of neutrino masses from Eqs. (12), while it will be equal to 0.005 (NH) and 0.05 eV (IH) at the mass values from Eqs. (13).

the NH case, the second and the third neutrino mass states are the quasi Dirac ones, while in the IH case they are correspondingly the first and the second mass neutrino states.

Then, the condition of the $\mu - \tau$ symmetry, i.e., the equality of $m_{\mu\mu}$ and m_{ee} results in the relation, which in the *NH* case reads as

$$|s_{12}^2 c_{23}^2 + 2 s_{12} c_{12} s_{23} c_{23} s_{13} e^{i\delta} + c_{12}^2 s_{23}^2 s_{13}^2 e^{2i\delta}| \\ = |s_{12}^2 s_{23}^2 - 2 s_{12} c_{12} s_{23} c_{23} s_{13} e^{i\delta} + c_{12}^2 c_{23}^2 s_{13}^2 e^{2i\delta}|\,, \quad (15)$$

while in the *IH* case it is as follows

$$s_{23}^2 = c_{23}^2\,. \quad (16)$$

Equation (15) permits one to determine the CP-violating phase δ, while Eq. (16) determines the angle θ_{23}. With using the data from Eqs. (4) we can obtain that $\delta \approx 100^\circ$ in the NH case, in contrast to the frequently used value of $\delta = 0^\circ$. However, the obtained value of δ is different also from the value $\delta = 180^\circ$ [1]. For this reason we consider that Eq. (15) is not fulfilled, that is the condition of the $\mu - \tau$ symmetry results in preferability of the IH case of the neutrino mass spectrum. In this case, it is possible to estimate the values of all the neutrino mass observables m_a, m_β and $m_{\beta\beta}$, if the minimal neutrino mass is of the order of λ. In eV, they are as follows

$$m_a \approx 0.034,\, m_\beta \approx 0.049,\, m_{\beta\beta} \approx 0.00005\,. \quad (17)$$

The dependences of $m_{\beta\beta}$ versus the minimal neutrino mass m_0 in the model of bimodal neutrinos are shown in Fig. 5, in the range of m_0 between 0 and 1 eV. The characteristic feature of these results is the absence of the bottom limitation for $m_{\beta\beta}$ in both NH and IH cases, because in the bimodal neutrinos case $m_{\beta\beta}$ depends only on just one term involving m_0, that is m_1 or m_3 in the NH or IH case, respectively.

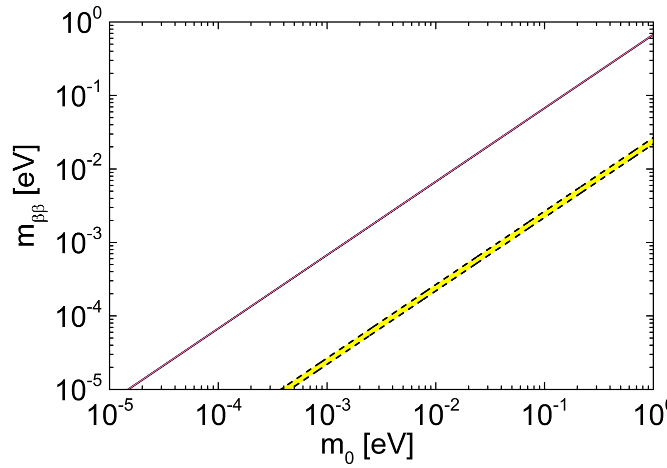


FIG. 5. (Color online) The $m_{\beta\beta}$ versus the minimal neutrino mass m_0 in the model of bimodal neutrinos at the oscillation parameters from Eqs. (4). The upper and lower narrow ranges restricted by the solid and the dashed lines correspond to the NH and IH cases, respectively.

VI. CONCLUSION

The properties of the neutrino are rather mysterious, and both intensive theoretical and experimental studies are necessary for determination of the nature and the characteristics of these elementary particles. The construction and development of adequate phenomenological models of neutrino, which generalize the SM in the neutrino sector is one of the ways to interpret and predict the experimental results, as well as to develop the future GUT. At present, one of the first-priority problem in both the theoretical and the experimental investigations is the ascertainment of the neutrino type, that is the Dirac type or the Majorana type. It may occur rather unexpected that, possibly, this problem has no unambiguous solution and the neutrino can be bimodal, as was considered in the present paper. The other important problems, which should be solved in the process of the theoretical and experimental investigations are the determination of the absolute mass scale, the characteristics of the CP violation of the neutrino, and the different correlations between the numerous neutrino parameters.

In the present paper, the possible values of the neutrino mass observables m_a, m_β and $m_{\beta\beta}$ were calculated on the basis of the most recent experimental data. It was found that the minimal mass neutrino range, where $m_{\beta\beta}$ vanishes is divided into two sub-ranges with limitation from the bottom between them for the $m_{\beta\beta}$ values in the case of the normal hierarchy. It takes place both for the neutrino mass spectrum in the process of neutrinoless double-beta decay with CP violation, and even in the intermediate range of m_0 for the decays with CP conservation. For the investigation of the neutrino properties, the $3+2+1$ phenomenological neutrino model with three active neutrinos and three sterile neutrinos was proposed. The model permits reducing the number of sterile neutrinos, if the model-independent experimental restrictions on their number will be established. In the framework of this model and with allowance for the recent experimental data, the values of the neutrino mass observables m_a, m_β and $m_{\beta\beta}$ were obtained, and also the estimations of masses $m_{si} \equiv M_i$ of the light sterile neutrinos [see Eqs. (14)] were made. As is known, for experimental determination of the observables m_a, m_β and $m_{\beta\beta}$, numerous experiments are currently carried out and planned, namely, the experiments on search the neutrinoless double-beta decay, on determination of the form of the tritium β decay spectrum, as well as the cosmological observations. The theoretical estimations of the values of m_a, m_β, $m_{\beta\beta}$ and m_{si} given above in this paper can be used for the interpretation and prediction of the results of these experiments.

ACKNOWLEDGMENTS

Authors are grateful to S. V. Semenov, Yu. S. Lutostansky, D. K. Nadezhin and I. V. Panov for useful discussions and remarks. This work was supported by the Russian Foundation for Basic Research (Grant No. 11-02-00882-a).

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